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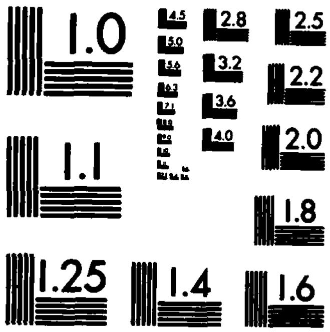

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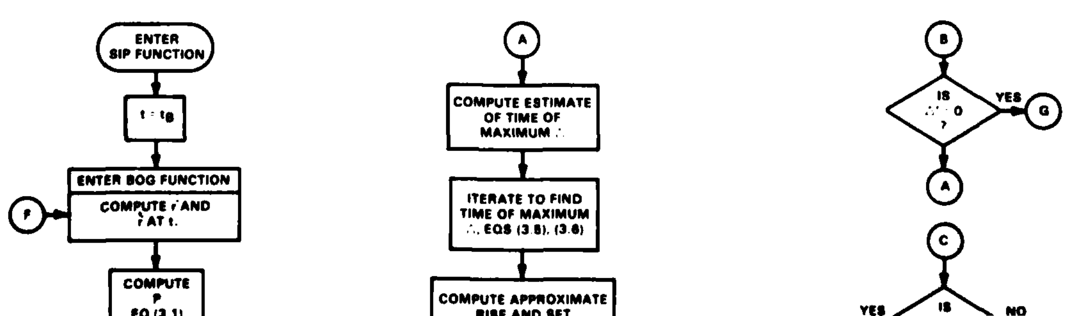

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A functional and mathematical description of the ground station alert predictor is presented. The alert predictor generates the times during which a satellite will be visible to a ground tracking station and provides ground antenna pointing information. The methods used to generate this information are computationally quite efficient so that long-term predictions can be done rapidly. Several new innovations are included in the algorithms: the effects of atmospheric drag upon satellites are taken into account, (continued on back)

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Block 20 continued.

and a satellite orbit adjust may be included during the prediction interval.

FOREWORD

This report presents a functional and mathematical description of the ground station alert predictor. The alert predictor generates the times during which a satellite will be visible to a ground tracking station and provides ground antenna pointing information. This is an invaluable tool that is useful for scheduling ground station operational activities. The methods used to generate this information are computationally quite efficient so that long-term predictions can be done rapidly. Several new innovations are included in the algorithms: the effects of atmospheric drag upon satellites are taken into account, and a satellite orbit adjust may be included during the prediction interval.

Released by:

O. F. BRAXTON, Head
Strategic Systems Department

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ILLUSTRATIONS

SECTION 1

INTRODUCTION

The primary function of the ALERT package is to predict the times during which a satellite will be visible at a ground tracking station and provide approximate ground antenna pointing information. The methods used to generate this information are computationally quite efficient, so that the in-view times and associated information for a given satellite and ground station can be rapidly predicted for fairly long prediction intervals (~ 30 days). An additional optional feature exists which accounts for the inclusion of a satellite orbit adjust during the prediction interval.

A functional overview of the ALERT generator is shown in Figure 1-1. As can be seen from this figure, the ALERT generator is comprised of four basic computational functions: the process flow supervisor (PFS), the Brouwer mean element converter (BMC), the satellite-station in-view predictor (SIP), and the Brouwer orbit generator (BOG). Also shown on this figure is the inter-function data flow. A detailed description of each of these functions is presented in the following sections.

As user supplied input, the ALERT generator requires the station and satellite and associated frequency information for which in-view periods are to be predicted, as well as the start (t_B) and stop (t_E) times of the time interval over which predictions are to be made. Also included in the input is the option selected by the user for the inclusion of the impact of orbit adjusts occurring during the prediction interval on in-view periods. In order to initiate its processing the ALERT generator requires the following additional data as shown on Figure 1-1: station latitude, longitude, and height; mean Brouwer elements and decay rates; and post orbit adjust epoch, inertial position and velocity vectors if the effect of an orbit adjust is to be included in the prediction span. The predicted inview periods and associated station antenna pointing geometry that were computed for the prediction interval are written to hardcopy at the end of the computational cycle.

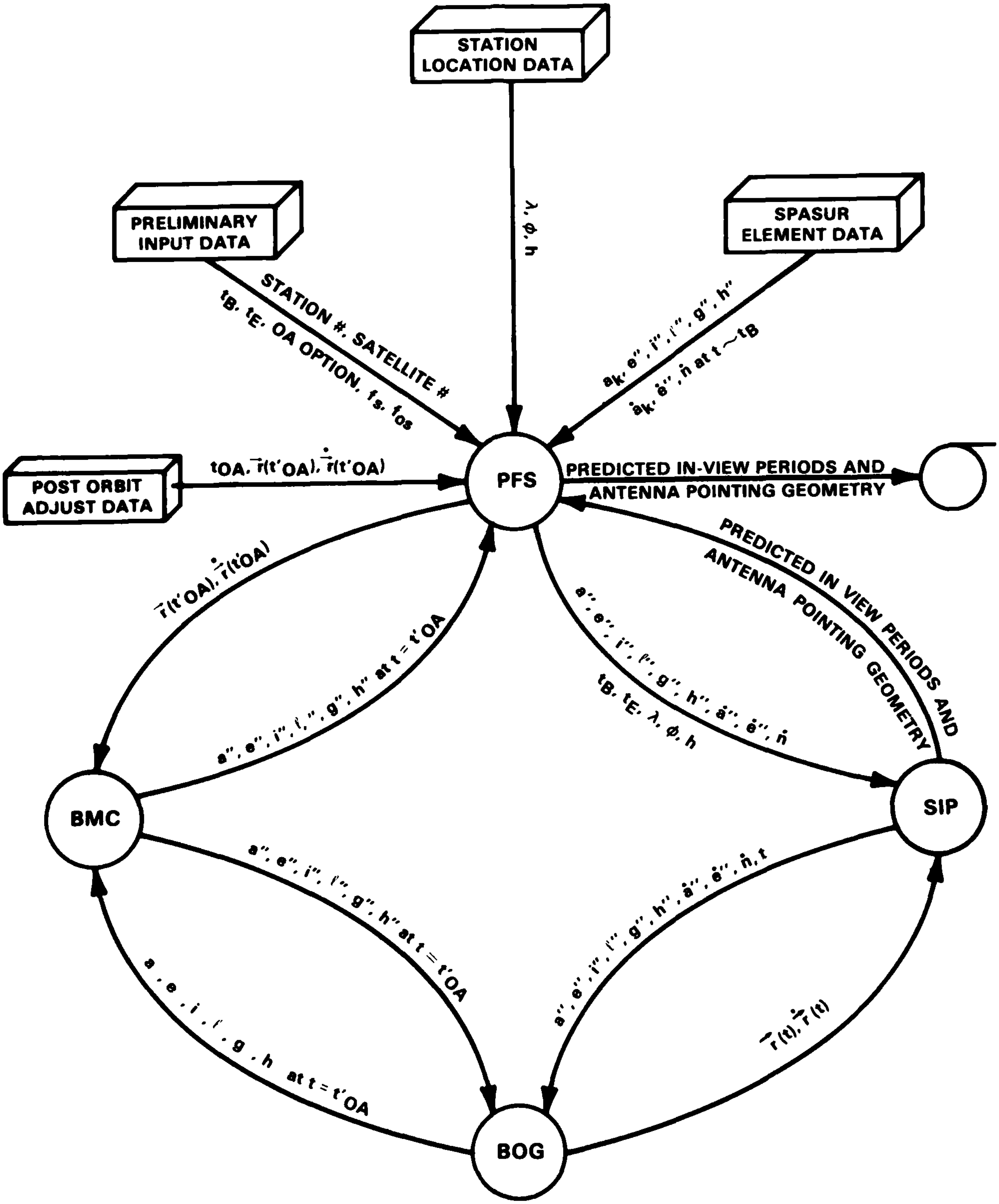

FIGURE 1-1. ALERT FUNCTIONAL OVERVIEW END DATA FLOW

SECTION 2

THE PROCESS FLOW SUPERVISOR (PFS)

2.1 FUNCTIONAL DESCRIPTION

The principal tasks performed by the PFS are to receive and retrieve data, direct processing flow, and output computed results. Specifically the PFS function:

(i) receives input data;
(ii) retrieves station coordinates;
(iii) retrieves initial SPASUR Brouwer mean elements and decay rates;
(iv) receives predicted orbit adjust times and post adjust vectors;
(v) cycles computations through BMC if required;
(vi) cycles computations through SIP; and
(vii) receives SIP predictions and outputs them.

The flow of the PFS function is presented in Figure 2-1.

It should be noted that tasks (iv) and (v) above are optional. Should the user elect to use one of the two orbit adjust options and an orbit adjust falls within the prediction span, the PFS retrieves the predicted orbit adjust time (t_{0A}) and the associated predicted post adjust position and velocity at a time t'_{0A} very near (but later than) t_{0A}, $\vec{r}(t'_{0A})$ and $\dot{\vec{r}}(t'_{0A})$. These osculating Cartesian vectors are passed to the BMC function where they are converted to Brouwer mean elements for the time t'_{0A}.

As noted above, the orbit adjust option can operate in two modes. If the orbit adjust option flag IOAOP is set to zero, the option is not used and SIP predictions are generated using only the initial element set obtained from the SPASUR data. However, if it is set to one or two, the retrieval and conversion of the post adjust vectors take place, as described above. When IOAOP is set to one, the prediction span is divided into two partitions using t'_{0A} as the boundary, and the initial Brouwer elements retrieved

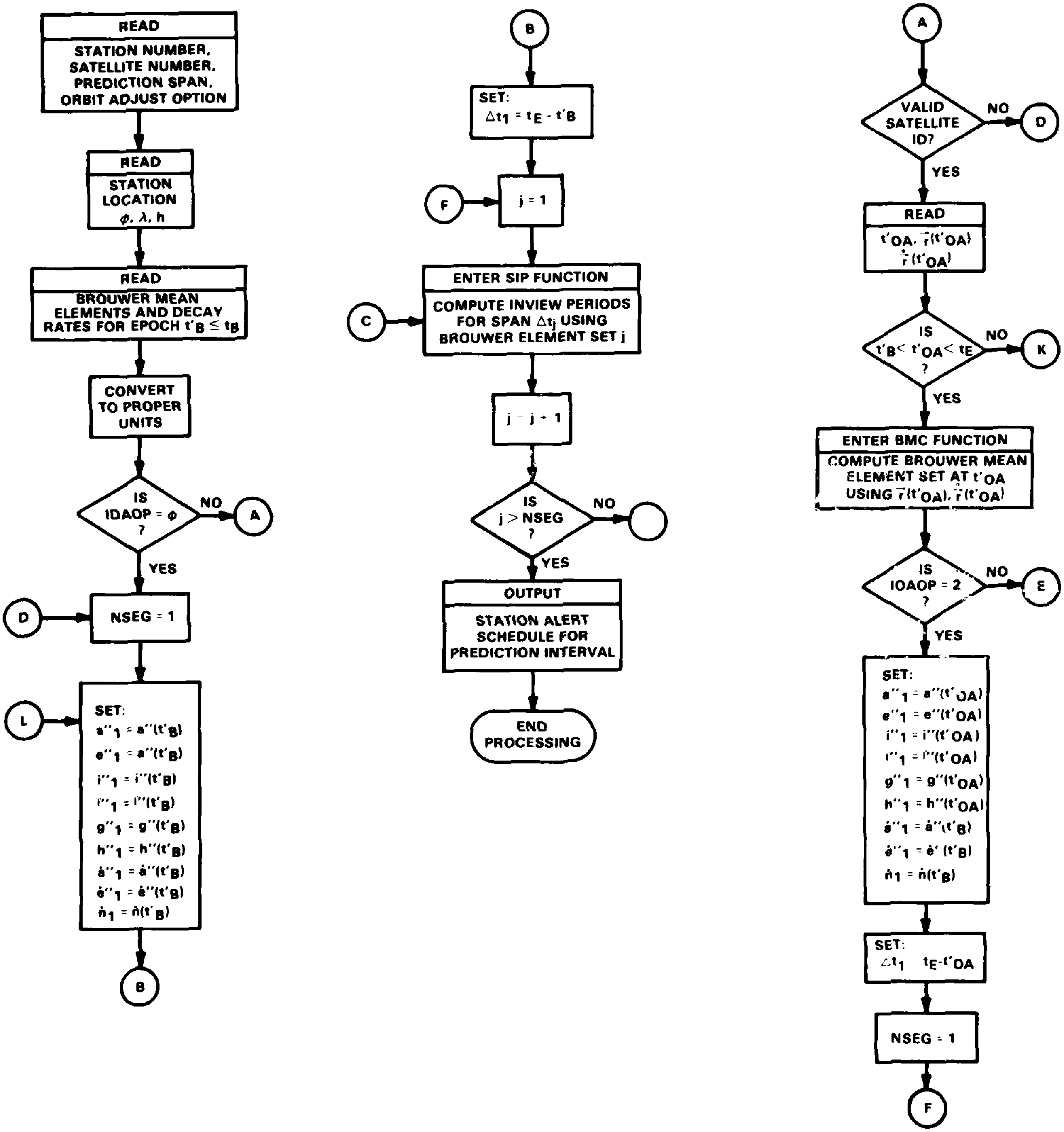

FIGURE 2-1. PROCESS FLOW SUPERVISOR LOGIC FLOW

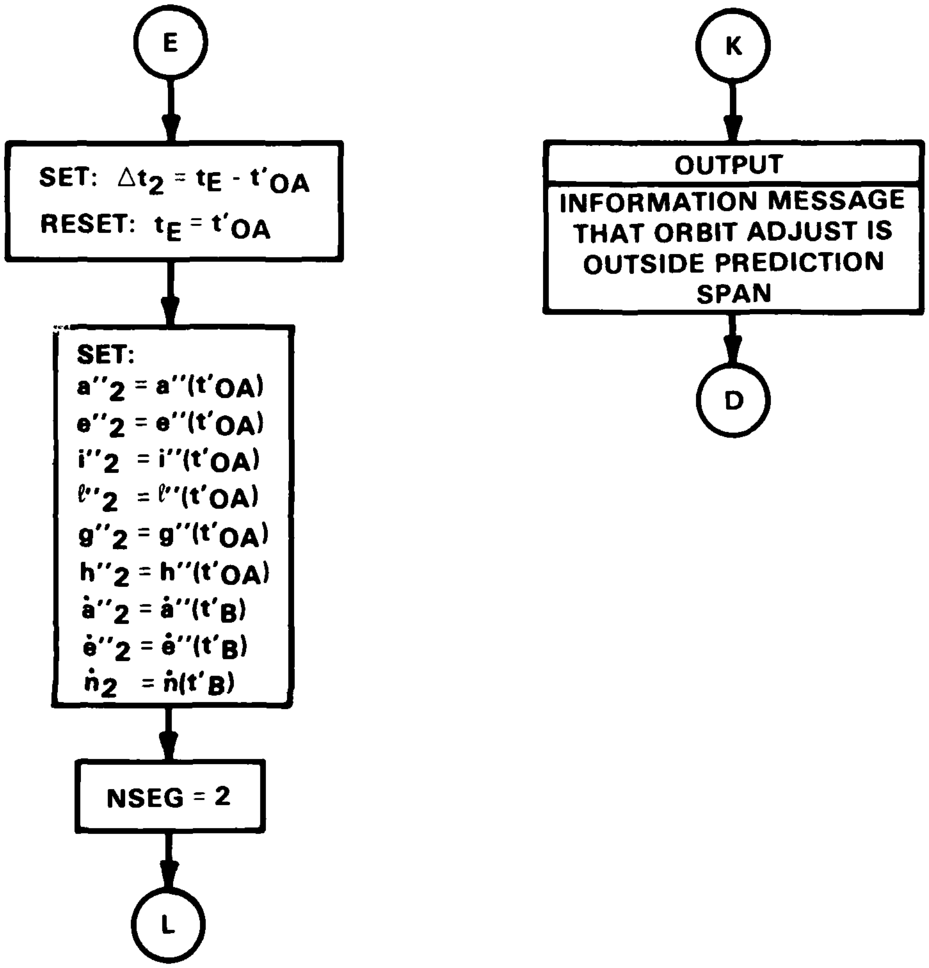

FIGURE 2-1. PROCESS FLOW SUPERVISOR LOGIC FLOW (CONTINUED)

2.1 FUNCTIONAL DESCRIPTION (con't)

by the PFS from the SPASUR element data are used for SIP in-view predictions until t'_{0A} is reached. For times later than t'_{0A} the post-orbit adjust Brouwer mean elements generated by the BMC function are used for SIP predictions. When IOAOP is set to two, SIP predictions are generated only for times later than t'_{0A} using the Brouwer element set generated by the BMC function.

2.2 SPECIAL FEATURES

Several points associated with the orbit adjust option are worthy of note. First, the orbit adjust options (IOAOP $\neq$ 0) may be selected only for those satellites which are supported by the PULSAR system[1]. If the option has been selected for a satellite other than those, the option is denied and processing continues as if the option had never been selected. Secondly, if the orbit adjust option has been validly selected, the PFS uses the decay rates obtained from the SPASUR element file for the epoch t'_B as the decay rates for times later than t'_{0A}.

The epoch t'_B above corresponds to the time of the Brouwer mean element set obtained from the SPASUR data nearest, but not later than, the start time of the prediction interval t_B, i.e. $t'_B \leq t_B$. If an orbit adjust occurs within the time interval between t'_B and t_B processing is still performed as described above. However, the alert schedule only for the time interval $[t_B, t_E]$ will be output.

---

[1] Parks, A.D. and Hicks, T.I., A Mathematical Description of the PULSAR Doppler Tracking Data Editor, NSWC/DL TR 82-391, Dahlgren, Virginia, 1982.

2.3 PROCESSING EQUATIONS

The semi-major axis read from the SPASUR data is the Kaula semi-major axis a_k expressed in earth radii. This is converted in the PFS to the Brouwer mean semi-major axis a" via the transformation

$$a'' = a_k \, a_e \left(\frac{1 + 2X}{1 - X} \right)^{2/3} , \tag{2.1}$$

where

$$X = \frac{3J_2(1 - 3/2 \sin^2 i'')}{4 \, a_k^2 \, (1 - e''^2)^{3/2}} . \tag{2.2}$$

In the above expressions a_e is the earth's semi-major axis; J_2 is a zonal harmonic gravitational constant; and i'' and e'' are the Brouwer mean inclination and eccentricity, respectively.

Similarly, the semi-major axis decay rate obtained from the SPASUR data is the time rate of change of the Kaula semi-major axis $\dot{a}_k$. This is converted in PFS to the time rate of change of the Brouwer mean semi-major axis $\dot{a}''$ through application of the expression

$$\dot{a}'' = \dot{a}_k \, a_e \left(\frac{1 + 2X}{1 - X} \right)^{2/3} + 2 \, a_k \, a_e \, \dot{X} \left[(1 + 2X)(1 - X)^5 \right]^{-1/3} , \tag{2.3}$$

where

$$\dot{X} = X \left[3\left(\frac{e''}{1 - e''^2} \right) \dot{e}'' - 2 \left(\frac{\dot{a}_k}{a_k} \right) \right] . \tag{2.4}$$

In the last expression it has been assumed that

$$(\dot{i}'') = 0. \tag{2.5}$$

Values for $\dot{e}''$ and the rate of change of the mean motion $\dot{n}$ are also obtained from the SPASUR data.

SECTION 3

THE STATION-SATELLITE INVIEW PREDICTOR (SIP)

3.1 FUNCTIONAL DESCRIPTION[2]

The station-satellite in-view times can be obtained by considering the vertical distance Δ of the satellite above a horizontal plane tangent to the earth at the station's location. An inview period is considered to begin or end when the satellite is in the horizontal station plane, i.e., when $\Delta = 0$. Since the earth rotates, the satellite will dip below the zero level and may not appear above the horizontal plane for many revolutions.

To initiate SIP processing, it must first be determined where the satellite is at time t_B with respect to the maximum points of the function describing Δ. Once this is done, the first maximum is approached in fractional parts of a satellite period P until a close proximity of the maximum Δ is reached. This is accomplished by using the signs and magnitudes of the Δ and its first and second time derivatives, Δ' and Δ'', to determine how far to step forward in time. When the maximum Δ has been approached sufficiently close, and it is greater than zero, the exact time of maximum Δ is obtained iteratively. This time is used to compute approximate rise and set times, which are then refined using an iterative method. This procedure is repeated to find successive maximum Δs and associated rise and set times until the end of the prediction interval t_E is exceeded.

For each in-view period geometric parameters are computed and stored. When the end of the prediction interval is exceeded, the in-view times and associated geometry computations are passed to the PFS for output. The process flow of SIP is presented in Figure 3-1.

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[2] Castro, H.E., A New Method for Finding Station Prediction Alerts, NSWC/DL TM K-1/63, Dahlgren, Virginia, 1963.

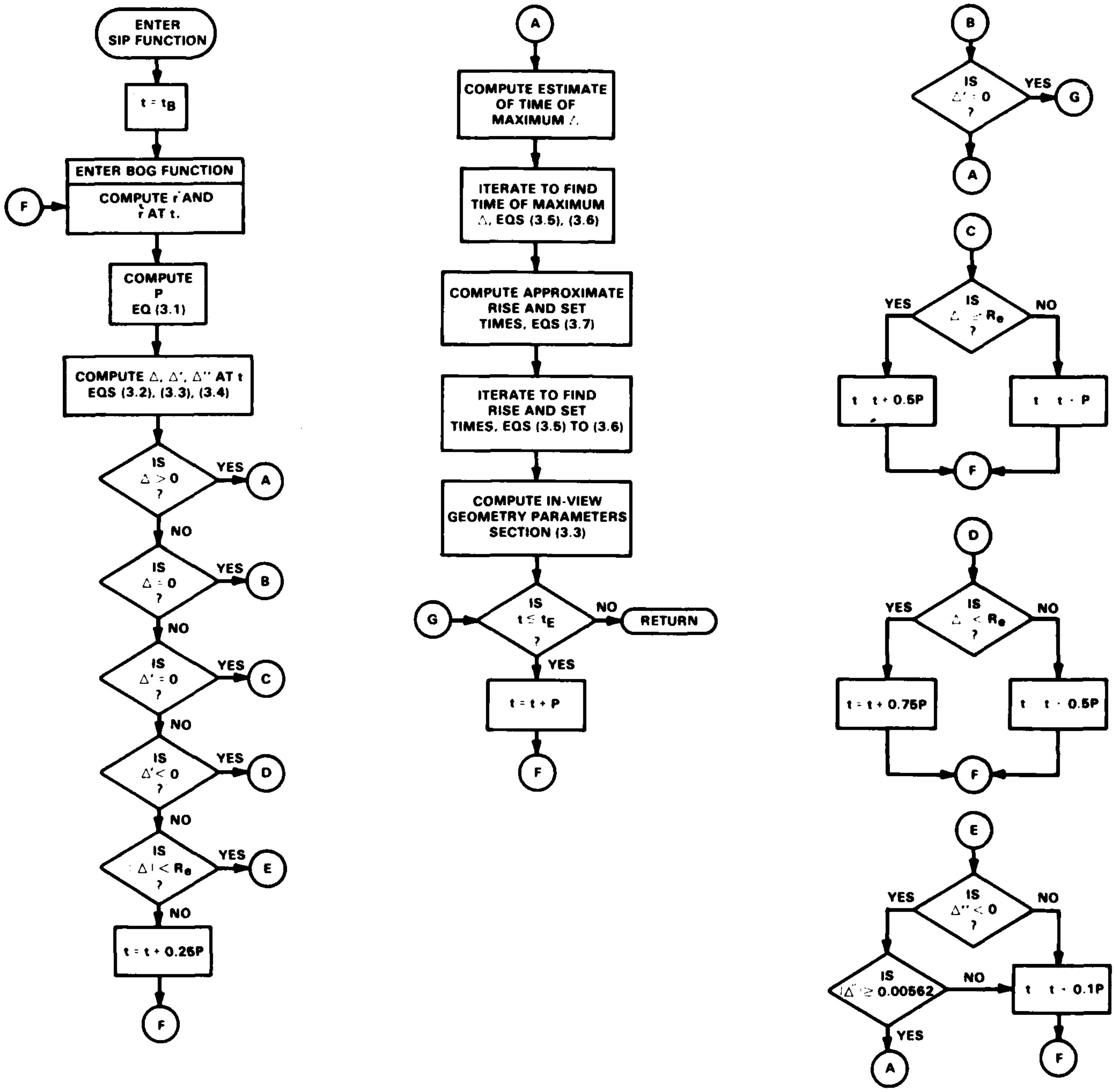

FIGURE 3-1. STATION-SATELLITE IN-VIEW PREDICTOR PROCESS FLOW

3.2 PROCESSING EQUATIONS

The orbital period P is computed using

$$P = \frac{2\pi}{\mu^{1/2}} a^{3/2} \tag{3.1}$$

where μ is the gravitational constant and a the semi-major axis.

The vertical distance Δ of a satellite from a horizontal plane tangent at a station location is given by:

$$\Delta = \cos\phi\,(x \cos\alpha + y \sin\alpha) + z \sin\phi - R_0 \quad , \tag{3.2}$$

where

$\alpha = \omega(t - t_{ve}) + \lambda$,

$R_0 = a_e (1-\varepsilon^2 \sin^2\phi)^{1/2} + h$,

ϕ = station geodetic latitude,

λ = station longitude,

ω = earth's rotation rate,

a_e = earth's equatorial radius,

t_{ve} = universal time of transit of vernal equinox,

ε = eccentricity of earth,

x, y, z = satellite inertial position components, and

h = height of station above reference ellipsoid.

3.2 PROCESSING EQUATIONS (con't)

The first and second time derivatives of Δ are given by

$$\Delta' = \cos\phi \left[(\dot{y} - \omega x) \sin\alpha + (\dot{x} + \omega y) \cos\alpha \right] + \dot{z} \sin\phi \ , \tag{3.3}$$

and

$$\Delta'' = \cos\phi \left[(\ddot{y} - 2\omega\dot{x} + \omega^2 y) \sin\alpha + (\ddot{x} + 2\omega\dot{y} - \omega^2 x) \cos\alpha \right] + \ddot{z} \sin\phi \ , \tag{3.4}$$

where

$\dot{x}, \dot{y}, \dot{z}$ = satellite inertial velocity components,

$$\ddot{x} = -\mu \frac{x}{r^3} \ ,$$

$$\ddot{y} = -\mu \frac{y}{r^3} \ , \text{ and}$$

$$\ddot{z} = -\mu \frac{z}{r^3} \ .$$

The expression required for the iterative solution for the time of maximum Δ, i.e. t_m, is given by

$$t_m^{(i+1)} = t_m^{(i)} - \frac{\Delta'(t_m^{(i)})}{\Delta''(t_m^{(i)})} \ , \tag{3.5}$$

where the superscripts in parentheses indicate t_m s obtained from the i^{th} iteration. Equation (3.5) is assumed to have converged when the tolerance condition

$$\left| t_m^{(i+1)} - t_m^{(i)} \right| < \tau_1 \ , \tag{3.6}$$

3.2 PROCESSING EQUATIONS (con't)

is satisfied. The first estimate of t_m to be used in equation (3.5) for $t_m^{(0)}$ is the value of t obtained prior to entering the A section of logic in Figure 3-1.

First estimates of the rise time t_r and set time t_s associated with the time of maximum Δ, t_m, are given by

$$t_r = t_m - \sqrt{\frac{-2\Delta}{\Delta''}} \quad \text{and} \quad t_s = t_m + \sqrt{\frac{-2\Delta}{\Delta''}} \quad . \tag{3.7}$$

These estimates are used to initiate iterative solutions for more exact rise and set times using equation (3.5). Convergence is assumed to have occurred when equation (3.6) is satisfied for both t_r and t_s.

3.3 IN-VIEW GEOMETRY PARAMETER COMPUTATIONS

The time of maximum Δ, i.e., t_m, is used to initiate the iterative solution for the time of closest approach t_{CA}, via the following expression:

$$t_{CA}^{(i+1)} = t_{CA}^{(i)} - \left(\frac{A}{\dot{A} - \dfrac{A^2}{|\vec{r} - \vec{r}_o|^2}} \right)_{t = t_{CA}^{(i)}} , \tag{3.8}$$

where

$$A = (x - a_1 \cos\alpha)(\dot{x} + a_1 \omega \sin\alpha) + (y - a_1 \sin\alpha)(\dot{y} - a_1 \omega \cos\alpha) + (z - b_1)\dot{z} , \tag{3.9}$$

$$\dot{A} = (x - a_1 \cos\alpha)(\ddot{x} + a_1 \omega^2 \cos\alpha) + (\dot{x} + a_1 \omega \sin\alpha)^2 + (y - a_1 \sin\alpha)(\ddot{y} + a_1 \omega^2 \sin\alpha) + (\dot{y} - a_1 \omega\cos\alpha)^2 + (z - b_1)\ddot{z} + \dot{z}^2 , \tag{3.10}$$

$$|\vec{r} - \vec{r}_o|^2 = (x - a_1 \cos\alpha)^2 + (y - a_1 \sin\alpha)^2 + (z - b_1)^2 , \tag{3.11}$$

3.2 PROCESSING EQUATIONS (con't)

$$a_1 = \left[a_e(1 - \varepsilon^2 \sin^2\phi)^{-\frac{1}{2}} + h\right] \cos\phi \quad , \tag{3.12}$$

and

$$b_1 = \left[a_e(1 - \varepsilon^2)\ (1 - \varepsilon^2 \sin^2\phi)^{-\frac{1}{2}} + h\right] \sin\phi \quad . \tag{3.13}$$

The azimuths A at t_r, t_{CA}, and t_s are computed using the equation:

$$A = \tan^{-1}\left\{\frac{-(x\sin\alpha - y\cos\alpha)}{-\sin\phi\ (x\cos\alpha + y\sin\alpha) + \cos\phi\left[(z + a_e\varepsilon^2 \sin\phi)/(1-\varepsilon^2 \sin^2\phi)^{\frac{1}{2}}\right]}\right\} . \tag{3.14}$$

If

$$|x\sin\alpha - y\cos\alpha| < \tau_2 \tag{3.15}$$

and

$$\left|\sin\phi\,(x\cos\alpha + y\sin\alpha) - \cos\phi\left[(z + a_e\varepsilon^2 \sin\phi)/(1 - \varepsilon^2 \sin^2\phi)^{\frac{1}{2}}\right]\right| < \tau_2, \tag{3.16}$$

then A is set to zero.

The Doppler frequency Δn (scaled to 100 mc and including the offset frequency) is computed at t_r and t_s using the relation:

$$\Delta n = \left(\frac{-f_s}{.2997928}\right) \left[(x-a_1\cos\alpha)(\dot{x} + a_1\omega\sin\alpha) - (y - a_1 \sin\alpha)(\dot{y} - a_1\omega\cos\alpha) + (z - b_1)\dot{z}\right] \cdot \left[(x - a_1 \cos\alpha)^2 + (y - a_1 \sin\alpha)^2 + (z - b_1)^2\right]^{-\frac{1}{2}}\left(\frac{100}{108}\right) + 100f_{os} \quad , \tag{3.17}$$

3.2 PROCESSING EQUATIONS (con't)

where f_s is the satellite frequency and f_{os} is the satellite off-set frequency.

The elevation E at t_{CA} is computed using the expression

$$E = \sin^{-1} \frac{\Delta}{|\vec{r} - \vec{r}_o|} \quad , \tag{3.18}$$

and the slant range ρ at t_{CA} is obtained from

$$\rho = [(x - a_1 \cos\alpha)^2 + (y - a_1 \sin\alpha)^2 + (z - b_1)^2]^{\frac{1}{2}} \quad . \tag{3.19}$$

SECTION 4

THE BROUWER ORBIT GENERATOR (BOG)

4.1 FUNCTIONAL DESCRIPTION

The Brouwer orbit generator function is used by the station alert prediction software package to supply predicted inertial position and velocity vectors for the satellite being processed. These vectors are used by the SIP function to generate station-satellite in-view periods. The BOG function is also used by the BMC function to convert osculating inertial Cartesian coordinates to mean Brouwer elements (it is assumed in this case that the satellites for which this is done have inclinations which are not in the neighborhood of the Brouwer-Lyddane singularity at ~63.5°).

Two methods are available for generating osculating elements in the BOG function. As long as the satellite inclination is not in the neighborhood of 63.5°, the Brouwer-Lyddane[3,4] theory modified to include drag effects is used to generate predicted osculating elements. If the inclination is sufficiently close to 63.5°, a Keplerian update method using secular terms only and no drag effects is applied to the mean elements. These updated elements are then used as the osculating elements in subsequent SIP calculations. In each case when BOG is being utilized by the SIP function, the osculating elements are transformed to osculating inertial Cartesian coordinates before returning to SIP. If BOG is being used by the BMC functions, no such transformation takes place. The BOG processing flow is shown in Figure 4-1.

---

[3] Brouwer, D., "Solution of the Problem of Artificial Satellite Theory Without Drag", The Astronomical Journal, Vol. 64, No. 1274, 1959, pp. 378 - 397.

[4] Lyddane, R. H., "Small Eccentricities or Inclinations in the Brouwer Theory of Artificial Satellites", The Astronomical Journal, Vol. 68, No. 8, 1963, pp. 555 - 558.

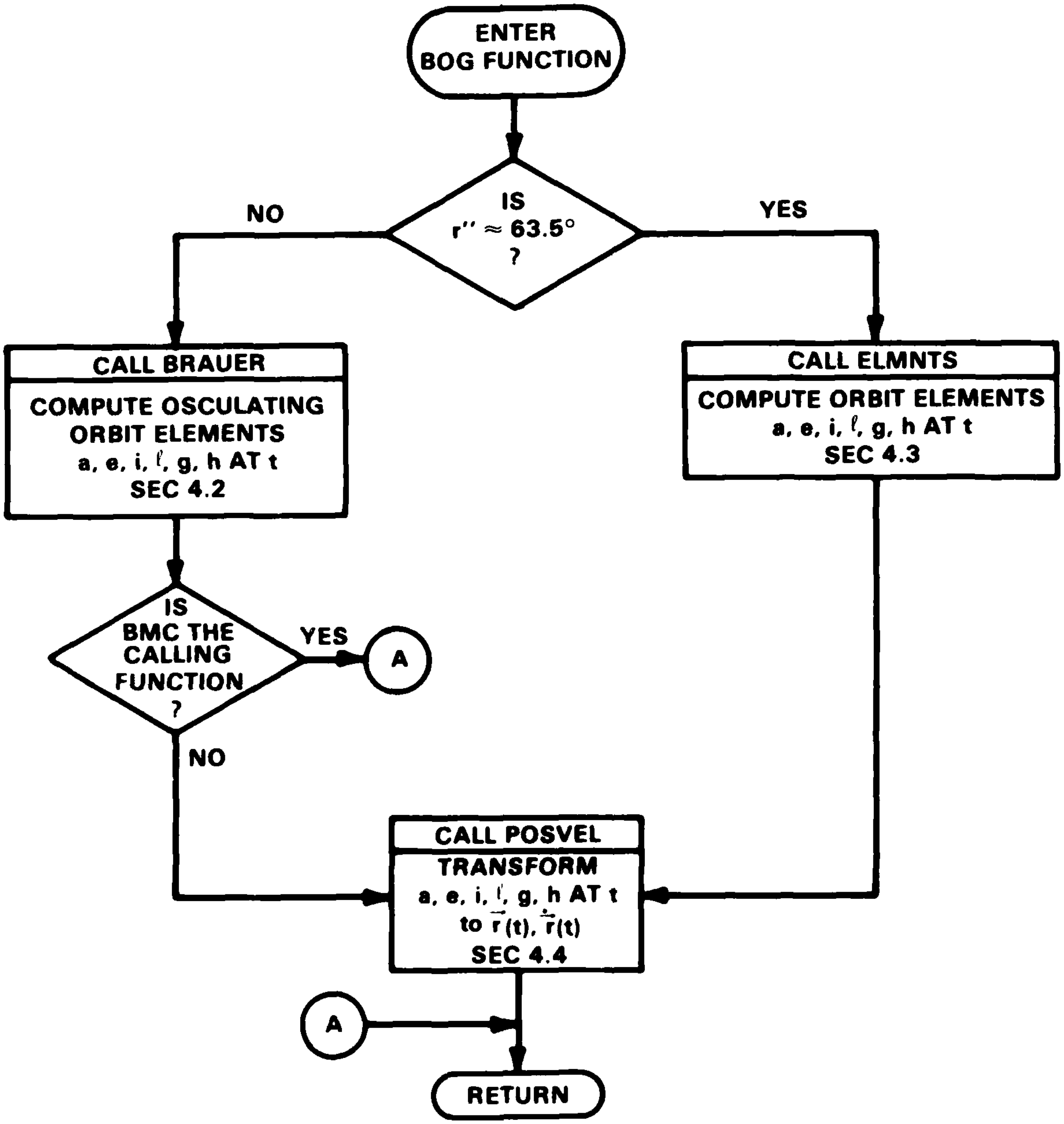

FIGURE 4-1. BROUWER ORBIT GENERATOR PROCESS FLOW

4.2 PROCESSING EQUATIONS FOR THE BROUWER-LYDDANE METHOD

The equations used to compute osculating orbital elements from mean Brouwer elements and decay rates are delineated in this section. First define the following:

$$\left.\begin{aligned}
\dot{a}'' &= \text{semi-major axis decay rate} \\
\dot{e}'' &= \text{eccentricity decay rate} \\
\dot{n} &= \text{time rate of change of mean motion} \\
t &= \text{time from epoch} \\
n_0 &= (\mu/a''^3)^{\frac{1}{2}} \\
\eta &= (1 - e''^2)^{\frac{1}{2}} \\
\theta &= \cos i'' \\
\gamma_2 &= \tfrac{1}{2}\, C_{20}\, a_e^2 / a''^2 \\
\gamma_2' &= \gamma_2 \eta^{-4} \\
\gamma_3' &= -C_{30}\, a_e^3\, a''^{-3} \eta^{-6} \\
\gamma_4' &= -3/8\, C_{40} a_e^4\, a''^{-4} \eta^{-8} \\
\gamma_5' &= -C_{50}\, a_e^5\, a''^{-5} \eta^{-10} \\
\alpha &= 1 - 5\theta^2 \\
\beta &= 1 - 11\theta^2 - 40\theta^4\, \alpha^{-1} \\
\gamma &= 1 - 3\theta^2 - 8\theta^4\, \alpha^{-1} \\
\delta &= 1 - 9\theta^2 - 24\theta^4\, \alpha^{-1} \\
\lambda &= 1 - 5\theta^2 - 16\theta^4\, \alpha^{-1} \quad ,
\end{aligned}\right\} \qquad (4.1)$$

where the C_{j0} (j = 2, 3, 4, 5) are the zonal harmonic gravitational expansion coefficients. Then the secular terms are computed from:

4.2 PROCESSING EQUATIONS FOR THE BROUWER-LYDDANE METHOD (con't)

$$\ell'' = n_0 t \left\{ 1 + \frac{3}{2}\gamma_2'\eta(3\theta^2 - 1) + \frac{3}{32}\gamma_2'^2\eta\left[-15 + 16\eta + 25\eta^2 + (30 - 96\eta - 90\eta^2)\theta^2 + (105 + 144\eta + 25\eta^2)\theta^4\right] + \frac{15}{16}\gamma_4'\eta e''^2\left[3 - 30\theta^2 + 35\theta^4\right]\right\} + \ell_0'' + \dot{n}t^2 \tag{4.2}$$

$$g'' = n_0 t \left\{ -\frac{3}{2}\gamma_2'\alpha + \frac{3}{32}\gamma_2'^2\left[-35 + 24\eta + 25\eta^2 + (90 - 192\eta - 126\eta^2)\theta^2 + (385 + 360\eta + 45\eta^2)\theta^4\right] + \frac{5}{16}\gamma_4'\left[21 - 9\eta^2 + (-270 + 126\eta^2)\theta^2 + (385 - 189\eta^2)\theta^4\right]\right\} + g_0'', \tag{4.3}$$

and

$$h'' = n_0 t \left\{ -3\gamma_2'\theta + \frac{3}{8}\gamma_2'^2\left[(-5 + 12\eta + 9\eta^2)\theta + (-35 - 36\eta - 5\eta^2)\theta^3\right] + \frac{5}{4}\gamma_4'(5 - 3\eta^2)\theta(3 - 7\theta^2)\right\} + h_0'' \quad . \tag{4.4}$$

The long period (dependent upon g'') terms are computed from:

$$\delta_1 e = \frac{35}{96}\frac{\gamma_5'}{\gamma_2'}e''^2\eta^2\lambda\sin i''\sin^3 g'' - \frac{1}{12}\frac{e''\eta^2}{\gamma_2'}(3\gamma_2'^2\beta - 10\gamma_4'\gamma)\sin^2 g'' - \frac{35}{128}\frac{\gamma_5'}{\gamma_2'}e''^2\eta^2\lambda\sin i''\sin g'' + \frac{1}{4}\frac{\eta^2}{\gamma_2'}\left[\gamma_3' + \frac{5}{16}\gamma_5'(4+3e''^2)\delta\right]$$

$$\sin i''\sin g'' + \frac{e''\eta^2}{24\gamma_2'}\left[3\gamma_2^{2\prime}\beta - 10\gamma_4'\gamma\right] \tag{4.5}$$

4.2 PROCESSING EQUATIONS FOR THE BROUWER-LYDDANE METHOD (con't)

$$\ell' + g' = g'' + \ell'' + \tfrac{1}{2}\left\{ \frac{1}{24\gamma_2'}\left[-3\gamma_2'^2\left\{2 + e''^2 - 11(2 + 3e''^2)\theta^2\right.\right.\right.$$

$$\left. - 40(2 + 5e''^2)\theta^4\alpha^{-1} - 400e''^2\theta^6\alpha^{-2}\right\}$$

$$\left. + 10\gamma_4'\left\{2 + e''^2 - 3(2 + 3e''^2)\theta^2 - 8(2 + 5e''^2)\theta^4\alpha^{-1} - 80e''^2\theta^6\alpha^{-2}\right\}\right]$$

$$\left. + \frac{\eta^3}{\gamma_2'}\left[\frac{\gamma_2'^2}{4}\beta - \frac{5}{6}\gamma_4'\gamma\right]\right\}\sin 2g'' + \left\{\frac{35}{384}\frac{\gamma_5'}{\gamma_2'}\eta^3 e''\lambda\sin i''\right.$$

$$+ \frac{35}{1152}\frac{\gamma_5'}{\gamma_2'}\left[\lambda\left\{-e''(3 + 2e''^2)\sin i'' + \frac{e''^3\theta^2}{\sin i''}\right\}\right.$$

$$\left.\left. + 2e''^3\theta^2\sin i''\left\{5 + 32\theta^2\alpha^{-1} + 80\theta^4\alpha^{-2}\right\}\right]\right\}\cos 3g''$$

$$+ \left\{-\frac{\gamma_3' e''\theta^2}{4\gamma_2'\sin i''} + \frac{5}{64}\frac{\gamma_5'}{\gamma_2'}\left[-e''\frac{\theta^2}{\sin i''}(4 + 3e''^2) + e''\sin i''\right.\right.$$

$$\left.(26 + 9e''^2)\right]\delta - \frac{15}{32}\frac{\gamma_5'}{\gamma_2'}e''\theta^2\sin i''(4 + 3e''^2)$$

$$(3 + 16\theta^2\alpha^{-1} + 40\theta^4\alpha^{-2}) + \tfrac{1}{4}\frac{\gamma_3'}{\gamma_2'}\sin i''.$$

$$\left(\frac{e''}{1+\eta^3}\right)\left[3 - e''^2(3 - e''^2)\right] + \frac{5}{64}\frac{\gamma_5'}{\gamma_2'}\eta^2\delta$$

$$\left.\left[\frac{e''(-32 + 81e''^4)}{4 + 3e''^2 + \eta(4 + 9e''^2)}\right]\sin i''\right\}\cos g'' \qquad (4.6)$$

4.2 PROCESSING EQUATIONS FOR THE BROUWER-LYDDANE METHOD (con't)

and

$$h' = h'' + \frac{35\,\gamma_5'\,e''^3\theta}{144\,\gamma_2'}\left\{\tfrac{1}{2}\lambda\sin^{-1}i'' + \sin i''\left[5 + 32\theta^2\alpha^{-1} + 80\theta^4\alpha^{-2}\right]\right\}$$

$$\sin^2 g''\cos g'' + \frac{e''^2\theta}{12\,\gamma_2'}\left\{-\,3\gamma_2'^2\left[11 + 80\theta^2\alpha^{-1} + 200\theta^4\alpha^{-2}\right]\right.$$

$$\left. + 10\gamma_4'\left[3 + 16\theta^2\alpha^{-1} + 40\theta^4\alpha^{-2}\right]\right\}\sin g''\cos g''$$

$$+\left\{-\frac{35\gamma_5'}{576\gamma_2'}\,e''^3\theta\left[\tfrac{1}{2}\lambda\sin^{-1}i'' + \sin i''\left(5 + 32\theta^2\alpha^{-1} + 80\theta^4\alpha^{-2}\right)\right]\right.$$

$$+\frac{e''\theta}{4\gamma_2'\sin i''}\left[\gamma_3' + \frac{5}{16}\gamma_5'\left(4 + 3e''^2\right)\delta + \frac{15}{8}\gamma_5'\left(4 + 3e''^2\right)\right.$$

$$\left.\left.\left(3 + 16\theta^2\alpha^{-1} + 40\theta^4\alpha^{-2}\right)\sin^2 i''\right]\right\}\cos g''\ . \qquad (4.7)$$

The short periodics (dependent upon E', f', ℓ'') are computed from:

$$a = \dot{a}''t + a'' - a''\frac{\gamma_2}{\eta^3}\left(3\theta^2 - 1\right) + \left[\frac{a''\gamma_2}{\left(1 - e''\cos E'\right)^3}\right]$$

$$\left[3\theta^2 - 1 + 3\sin^2 i''\cos\left(2g'' + 2f'\right)\right] \qquad (4.8)$$

$$e = e'' + \dot{e}''t + \delta_1 e + \frac{\eta^2\gamma_2}{2}\left\{\frac{3\theta^2 - 1}{\eta^6}\left[\frac{e''}{1+\eta^3}\left\{3 - e''^2\left(3 - e''^2\right)\right\}\right.\right.$$

$$\left. + \left\{3 + e''\cos f'\cdot\left(3 + e''\cos f'\right)\right\}\cos f'\right] + \frac{3\left(1 - \theta^2\right)}{\eta^6}$$

$$\left.\left[e'' + \left\{3 + e''\cos f'\left(3 + e''\cos f'\right)\right\}\cos f'\right]\cos\left(2f' + 2g''\right)\right\}$$

4.2 PROCESSING EQUATIONS FOR THE BROUWER-LYDDANE METHOD (con't)

$$- \frac{\eta^2 \gamma_2{}'}{2} (1 - \theta^2) \left[3\cos(2g'' + f') + \cos(2g'' + 3f') \right] \tag{4.9}$$

$$i = i'' - \frac{e''\theta}{\eta^2 \sin i''} \quad \delta_1 e + e''\gamma_2{}'\theta \sin i'' \sin f' \sin(2f' + 2g'')$$

$$+ 2e''\gamma_2{}'\theta \sin i'' \cdot \cos f' \cos(2f' + 2g'') + \frac{3}{2}\gamma_2{}'\theta \sin i'' \cos(2f' + 2g'') \tag{4.10}$$

$$g + \ell = g' + \ell' + \frac{\gamma_2{}'}{4} \Bigg\{ - 6\alpha(f' - \ell'' + e'' \sin f') + (3 - 5\theta^2)$$

$$\left[3\sin(2f' + 2g'') + 3e'' \sin(2g'' + f') + e'' \sin(2g'' + 3f') \right] \Bigg\}$$

$$+ \frac{e''\eta^2\gamma_2{}'}{4(1 + \eta)} \Bigg\{ 2(3\theta^2 - 1)(\sigma + 1) \sin f' + 3(1 - \theta^2)$$

$$\left[(1 - \sigma) \sin(2g'' + f') + (\sigma + 1/3) \sin(2g'' + 3f') \right] \Bigg\} \tag{4.11}$$

4.2 PROCESSING EQUATIONS FOR THE BROUWER-LYDDANE METHOD (con't)

$$h = h' + \left[2e''\gamma_2'\theta \cos f' + \frac{3}{2}\gamma_2'\theta\right] \sin(2g'' + 2f')$$

$$- e''\gamma_2'\theta \sin f' \cos(2f' + 2g'') - 3\gamma_2'\theta\,(f' - \ell'' + e''\sin f') \tag{4.12}$$

and

$$e\delta\ell = \frac{1}{2}\,\frac{e''\eta^3}{\gamma_2'}\left\{\frac{1}{4}\gamma_2'\beta - \frac{5}{6}\gamma_4'\gamma\right\}\sin 2g''$$

$$-\left\{\frac{1}{4}\,\frac{\gamma_3'}{\gamma_2'}\eta^3 \sin i'' + \frac{5}{64}\,\frac{\gamma_5'}{\gamma_2'}\eta^3 \sin i''(4 + 9e''^2)\delta\right\} \cdot$$

$$\cos g'' + \frac{35}{384}\,\frac{\gamma_5'}{\gamma_2'}\,\eta^3 e''^2\lambda \sin i'' \cos 3g''$$

$$- \frac{1}{4}\gamma_2'\eta^3\left\{2(3\theta^2 - 1)(\sigma + 1)\sin f' + 3(1 - \theta^2)(1 - \sigma)\sin(2g'' + f')\right.$$

$$\left. + \left(\sigma + \frac{1}{3}\right)\sin(2g'' + 3f')\right\}, \tag{4.13}$$

where

$$\sigma = \left(\frac{\eta}{1 - e''\cos E'}\right)^2 + \left(\frac{1}{1 - e''\cos E'}\right), \tag{4.14}$$

4.2 PROCESSING EQUATIONS FOR THE BROUWER-LYDDANE METHOD (con't)

the eccentric anomaly E' is obtained from a Newton-Raphson iteration upon the Kepler equation

$$E' - e'' \sin E' = \ell'' \quad , \tag{4.15}$$

and the true anomaly f' is found from

$$\sin f' = \frac{\eta \sin E'}{1 - e'' \cos E'} \quad , \tag{4.16}$$

$$\cos f' = \frac{\cos E' - e''}{1 - e'' \cos E'} \quad . \tag{4.16}$$

The final osculating values for a, i, and h are computed from equations (4.8), (4.10), and (4.12), respectively. Equations (4.2), (4.9), (4.11) and (4.13) are used to calculate final osculating values for ℓ, g, and e for the following relations:

$$A = e \cos \ell'' - e\delta\ell \sin \ell'' \quad , \tag{4.17}$$

$$B = e \sin \ell'' + e\delta\ell \cos \ell'' \quad , \tag{4.18}$$

$$\ell = \tan^{-1} (B/A) \quad , \tag{4.19}$$

$$g = (\ell + g) - \ell \quad , \tag{4.20}$$

and

$$e = (A^2 + B^2)^{\frac{1}{2}} \quad . \tag{4.21}$$

4.3 PROCESSING EQUATIONS FOR THE KEPLERIAN UPDATE METHOD

The Keplerian update method treats a, e, and i as constants and updates ℓ, g, and h using only their secular variations. The updated elements are given by:

$$\left.\begin{aligned} a &= a'' \\ e &= e'' \\ i &= i'' \end{aligned}\right\} , \qquad (4.22)$$

$$\ell = n_0 t \left[1 + \frac{3}{2} \gamma_2{}' \eta \left(3 \cos^2 i'' - 1 \right) \right] + \ell_0'' , \qquad (4.23)$$

$$g = \frac{3}{2} n_0 \gamma_2{}' t \left(5 \cos^2 i'' - 1 \right) + g_0'' , \qquad (4.24)$$

and

$$h = - 3 n_0 \gamma_2{}' t \cos i'' + h_0'' . \qquad (4.25)$$

The constants appearing in the last three equations are defined in equations (4.1).

4.4 PROCESSING EQUATIONS FOR TRANSFORMING ORBITAL ELEMENTS TO CARTESIAN COORDINATES

The SIP function uses the inertial Cartesian position and velocity vectors to compute inview periods. The inertial position and velocity are obtainable from the osculating orbital elements generated via the methods described above through application of the following transformations:

$$\vec{r} = a \underset{\sim}{\mathcal{O}} \begin{pmatrix} \cos E - e \\ \sqrt{1 - e^2} \sin E \end{pmatrix} , \qquad (4.26)$$

4.4 PROCESSING EQUATIONS FOR TRANSFORMING ORBITAL ELEMENTS TO CARTESIAN COORDINATES

and

$$\dot{\vec{r}} = \frac{(\mu a)^{\frac{1}{2}}}{a(1 - e\cos E)} \; \underset{\sim}{\mathcal{O}} \begin{pmatrix} -\sin E \\ \\ \sqrt{1 - e^2}\,\cos E \end{pmatrix} , \qquad (4.27)$$

where

$$\underset{\sim}{\mathcal{O}} = \begin{pmatrix} \cos h\cos g - \sin h\cos i\sin g & -\cos h\sin g - \sin h\cos i\cos g \\ \cos g\sin h + \cos h\cos i\sin g & \cos h\cos i\cos g - \sin h\sin g \\ \sin i\sin g & \sin i\cos g \end{pmatrix} . \qquad (4.28)$$

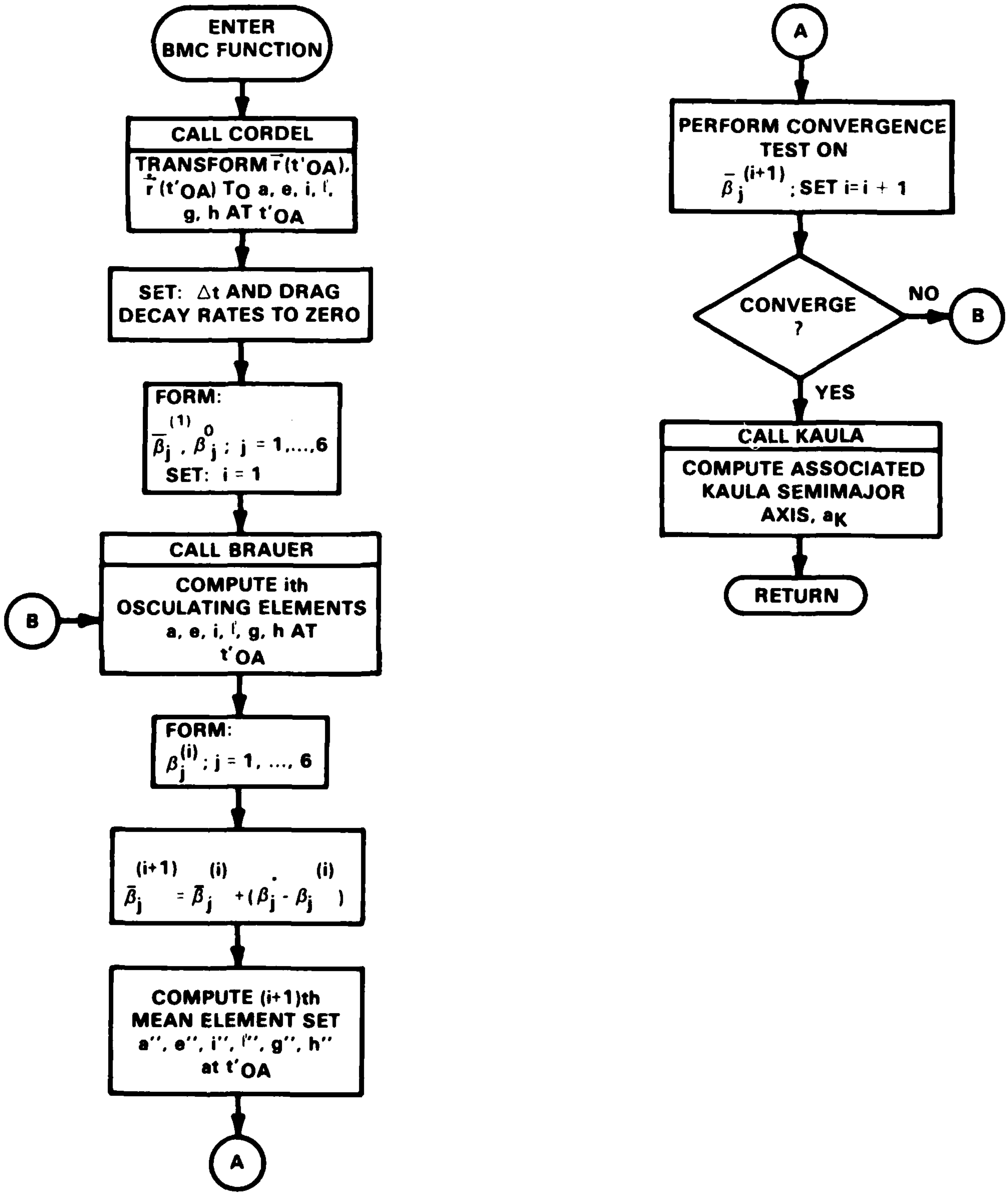

FIGURE 5-1. Process Flow of the Brouwer Mean Element Converter

SECTION 5

THE BROUWER MEAN ELEMENT CONVERTER (BMC)

5.1 FUNCTIONAL DESCRIPTION

The primary tasks performed by the BMC are to receive post orbit adjust inertial Cartesian position and velocity vectors at t'_{0A}, i.e., $\vec{r}(t'_{0A})$ and $\dot{\vec{r}}(t'_{0A})$, and:

(i) transform the osculating inertial Cartesian components to osculating Brouwer orbital elements;

(ii) iteratively solve for the Brouwer mean element set associated with the osculating inertial Cartesian components.

The iterative solution mentioned in (ii) above utilizes the BRAUER subroutine of the BOG function. The process flow of the BMC function is shown in Figure 5-1.

5.2 PROCESSING EQUATIONS

The post-orbit adjust osculating inertial Cartesian vectors $\vec{r}(t'_{0A}) = (x, y, z)$ and $\dot{\vec{r}}(t'_{0A}) = (\dot{x}, \dot{y}, \dot{z})$ are transformed to osculating Brouwer orbital elements using the following relationships:

$$a = \left[\frac{2}{|\vec{r}|} - \frac{|\dot{\vec{r}}|^2}{\mu} \right]^{-1} , \qquad (5.1)$$

$$e = \left\{ \frac{(\vec{r} \cdot \dot{\vec{r}})^2}{\mu a} + \left[\frac{|\vec{r}| \; |\dot{\vec{r}}|^2}{\mu} - 1 \right]^2 \right\}^{\frac{1}{2}} , \qquad (5.2)$$

$$i = \tan^{-1} \left\{ \frac{\left[(y\dot{z} - z\dot{y})^2 + (z\dot{x} - x\dot{z})^2 \right]^{\frac{1}{2}}}{x\dot{y} - y\dot{x}} \right\} , \qquad (5.3)$$

5.2 PROCESSING EQUATIONS (con't)

$$\ell = \tan^{-1}\left\{\frac{(x\dot{y} - y\dot{x})\, y}{X\,|x\dot{y} - y\dot{x}|}\right\} , \qquad (5.4)$$

$$h = \tan^{-1}\left\{\frac{y\dot{z} - z\dot{y}}{x\dot{z} - z\dot{x}}\right\} ,$$

and

$$g = \tan^{-1}\left\{\frac{z|\vec{r}\times\dot{\vec{r}}|\left[\frac{|\vec{r}||\dot{\vec{r}}|^2}{\mu} - 1 - e^2\right] + \left[x(z\dot{x} - x\dot{z}) - y(y\dot{z} - z\dot{y})\right]\sqrt{1 - e^2}\,\frac{\vec{r}\cdot\dot{\vec{r}}}{(\mu a)^{\frac{1}{2}}}}{z|\vec{r}\times\dot{\vec{r}}|\sqrt{1 - e^2}\,\frac{\vec{r}\cdot\dot{\vec{r}}}{(\mu a)^{\frac{1}{2}}} - \left[x(z\dot{x} - x\dot{z}) - y(y\dot{z} - z\dot{y})\right]\left[\frac{|\vec{r}||\dot{\vec{r}}|^2}{\mu} - 1 - e^2\right]}\right\} . \qquad (5.5)$$

The iterative technique used to find the mean Brouwer elements from the associated osculating elements is similar to that described by Walter[5]. However, a nonsingular orbital element set devised by Cohen and Hubbard[6] is used here to provide computational stability for near circular orbits. This nonsingular mean element set, represented by $\overline{\beta}_j$, where

[5] Walter, H.G., "Conversion of Osculating Elements into Mean Elements", The Astronomical Journal, Vol. 72, No. 8, 1967, pp. 994 - 997.

[6] Cohen, C.J. and Hubbard, E.C., "A Nonsingular Set of Orbit Elements", The Astronomical Journal, Vol. 67, No. 1, 1962, pp. 10 - 15.

5.2 PROCESSING EQUATIONS (con't)

$$\bar{\beta}_j = \begin{cases} p^{\frac{1}{4}} \cos\left(\frac{i''}{2}\right) \cos\left(\frac{h'' + g'' + \ell''}{2}\right) & , \quad j = 1 \\ p^{\frac{1}{4}} \sin\left(\frac{i''}{2}\right) \cos\left(\frac{h'' - g'' - \ell''}{2}\right) & , \quad j = 2 \\ p^{\frac{1}{4}} \sin\left(\frac{i''}{2}\right) \sin\left(\frac{h'' - g'' - \ell''}{2}\right) & , \quad j = 3 \\ p^{\frac{1}{4}} \cos\left(\frac{i''}{2}\right) \sin\left(\frac{h'' + g'' + \ell''}{2}\right) & , \quad j = 4 \\ e'' \cos \ell'' & , \quad j = 5 \\ -e'' \sin \ell'' & , \quad j = 6 \end{cases} \tag{5.6}$$

is obtained from the iterative process executed according to the scheme

$$\bar{\beta}_j^{(i+1)} = \bar{\beta}_j^{(i)} + (\beta_j^{\circ} - \beta_j^{(i)}) \quad , \quad j = 1, 2, \ldots, 6. \tag{5.7}$$

The p in equations (5.6) is the mean semi-latus rectum. The β_j° in the last equation are the initial nonsingular osculating elements, and the $\beta_j^{(i)}$ are the osculating nonsingular elements formed using the osculating elements obtained from BRAUER on the i^{th} iteration and generated by using the Brouwer mean elements obtained from $\bar{\beta}_j^{(i)}$. This procedure is repeated on each $\bar{\beta}_j^{(i)}$ until the following convergence conditions are satisfied:

$$|\bar{\beta}_j^{(i+1)} - \bar{\beta}_j^{(i)}| < \varepsilon_j \quad , \quad j = 1, 2, \ldots, 6, \tag{5.8}$$

5.2 PROCESSING EQUATIONS (con't)

where the ε_j are small numbers. When this condition is satisfied, then

$$\overline{\beta}_j = \overline{\beta}_j^{(i+1)} \quad , \quad j = 1, 2, 3, \ldots, 6, \tag{5.9}$$

and

$$p = \left(\sum_{j=1}^{4} \overline{\beta}_j^2 \right)^2 \quad , \tag{5.10}$$

$$e'' = (\overline{\beta}_5^2 + \overline{\beta}_6^2)^{1/2} \quad , \tag{5.11}$$

$$a'' = p/(1 - e''^2) \quad , \tag{5.12}$$

$$i'' = 2 \tan^{-1} \left\{ \frac{(\overline{\beta}_2^2 + \overline{\beta}_3^2)^{1/2}}{(\overline{\beta}_1^2 + \overline{\beta}_4^2)^{1/2}} \right\}, \tag{5.13}$$

$$\ell'' = \tan^{-1} (-\overline{\beta}_6/\overline{\beta}_5) \quad , \tag{5.14}$$

$$h'' = \tan^{-1} (\overline{\beta}_3/\overline{\beta}_2) + \tan^{-1} (\overline{\beta}_4/\overline{\beta}_1) \quad , \tag{5.15}$$

and

$$g'' = 2 \tan^{-1} (\overline{\beta}_4/\overline{\beta}_1) - (\ell'' + h'') \quad . \tag{5.16}$$

5.2 PROCESSING EQUATIONS (con't)

For completeness, the BMC function computes the Kaula semi-major axis a_K associated with the post-orbit adjust mean semi-major axis $a''(t'_{0A})$. This is generated recursively, using the Newton-Raphson method, i.e.,

$$a_K^{(i)} = g\left(a_K^{(i-1)}\right) \quad , \tag{5.17}$$

where

$$g(a_K) = a_K - \left\{ \frac{a_K^7 + 4\delta a_K^5 - \gamma a_K^4 + 4\delta^2 a_K^3 + 2\,\delta\gamma a_K^2 - \gamma\delta^2}{7a_K^6 + 20\delta a_K^4 - 4\gamma a_K^3 + 12\delta^2 a_K^2 + 4\delta\gamma a_K} \right\} \quad . \tag{5.18}$$

In the last equation,

$$\delta = \frac{3J_2(1 - \frac{3}{2}\sin^2 i'')}{4(1 - e''^2)^{3/2}} \tag{5.19}$$

and

$$\gamma = \left({}^{a''}/_{a_e}\right)^3 \quad . \tag{5.20}$$

This iterative procedure is continued until

$$\left|a_K^{(i+1)} - a_K^{(i)}\right| < 10^{-5} \quad . \tag{5.21}$$

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